Example where left coset $\neq$ right coset

(Originally asked on Discord)

Let G be the group of bijections on $\mathbb{R}^+$ under composition, let H be the subgroup generated by the doubling map $x \mapsto 2x$, and let g be the squaring map $x \mapsto x^2$. Then gH consists of maps of the form $x \mapsto (2^n x)^2 = 2^{2n}x^2$, while Hg consists of maps of the form $x \mapsto 2^n x^2$. Thus $gH \subset Hg$.

Way Yan
26/9/2021